DYNAMIC TRANSIENT MODE OF SHEAR STRAIN OF AN ELASTIC VISCOPLASTIC MEDIUM IN A LONG CHANNEL

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UDC 532.135

A transient regime with insignificant travel of a nondestructed structure along the channel is possible due to the excess of dynamic loads over static ones in elastoviscoplastic medium. A short-time slip of the elastic frame relative to the channel walls is bound up with propagation of loading and unloading waves in a nondestructed structure.

Plastic disperse systems of the aqueous bentonite suspension type, and consistent lubricants are capable of forming coagulation thixotropic structures [1]. The structural-mechanical properties of the spatial skeleton govern the rheological properties of a disperse system. The behavior of plastic structurized media with inelastic dispersion phase is studied experimentally in [2, 3]. Analysis of the rheological dependences permitted extraction of the range of stress variation corresponding to the different state of the structure. Only elastic strains completely reversible under unloading are developed for stresses below a certain elastic limit ($\tau < \tau_k$). Exceeding the yield point results in fracture in the structural skeleton and plastic flow. Two domains of variation of the shear stress are established in [2, 3]: the creep domain ($\tau_k < \tau < \tau_r$) and the plastic flow domain ($\tau_r < \tau < \tau_b$). Only local fractures in the structure that succeed in being restored correspond to the creep state. The plastic flow is characterized by avalanche fracture of the structural skeleton and is a flow of structurized dispersion with an ultimately destroyed structure. Passage of the system to purely Newtonian flow with completely destroyed coagulation-thixotropic relations corresponds to stresses $\tau > \tau_b$. Therefore, qualitatively different nature of the dispersion strain corresponds to stresses of different intensities. Under real flow conditions, for instance, in a channel where the stress distribution is inhomogeneous and can change with time, zones of elastic strain, creep, plastic flow can be separated out, and finally, Newtonian flow for very large shear stresses (this latter zone is absent for the moderate stresses being considered in this paper). It is important to note that propagation of perturbations in the elastic domain is of wave nature, consequently, a loading wave can arrive at this point that will cause destruction of the structure and its flow, and then an unloading wave that will restore the destroyed bonds and cut off the flow. Imposition of a longitudinal pressure gradient in a plane channel without causing destruction of the structural channel results in propagation of a transverse elastic stress wave of triagnular profile with maximal value $\tau_{max} = H(\partial p/\partial z)_0$ (H is the channel width, and $(\partial p/\partial z)_0$ is the pressure gradient). If the loading conditions are such that $\tau_k < H(\partial p/\partial z)_0$, then destruction of the structure starts at a certain time at the channel wall and is propagated to the channel center at the elastic wave shear velocity. The unloading wave in the elastic domain diminishes the stress and causes contraction of the flow zone and also total disappearance of the destroyed structure zone for a sufficiently high intensity. In the first case the transient is terminated when a stationary velocity and stress distribution is established in the plastic flow domain, and in the second when the appearance of a flow zone ceases and a vibrational mode of reversible strain of the structural skeleton is established. The transient of the second kind is called dynamic [4]. The purpose of this paper is indeed to study the strain mode of a disperse medium of dynamic type. Its principal feature is the fluctuating nature of the near-wall flow.

A mathematical model was formulated earlier in [4, 5] and a numerical solution was obtained for the shear strain problem of an elastic viscoplastic medium in a long channel as the pressure gradient varies. In this paper a qualitative analysis is performed of the

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A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 3, pp. 448-453, September, 1988. Original article submitted September 9, 1986.

0022-0841/88/5503-1025$12.50

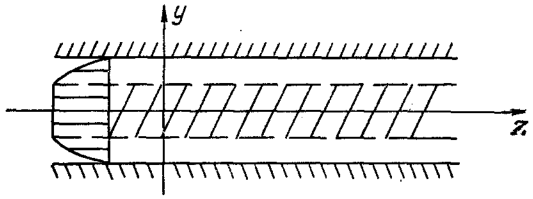

Fig. 1. Strain scheme.

dynamic transient mode on the basis of an asymptotic solution. The solution, found in the form of a d'Alembert wave, shows how reconstruction of the stress profile in the structure is related to the appearance and disappearance of near-wall flow zones with a destroyed structure. The single assumption used to obtain the analytic solution concerns the width of the flow zone with the whole channel width ($h(t) \ll H$). As will be shown below, this condition is stisfied, if, firstly, $\varepsilon = (H(\partial p/\partial z)_0 - \tau_k)/(H(\partial p/\partial z)_0/2) \ll 1$ is a relatively small increase in the maximal elastic stress over the value of the yield point, and secondly, $T_b/T_y \ll 1$, the characteristic time of propagation of a viscous shear wave in a zone with destroyed structure $T_b = \rho h^2/\eta$ is less than the elastic in the undestroyed structure $T_y = H\sqrt{\rho/G}/2$. A model taking account of the finite strains of the structural skeleton and assuming nearby values of the elastic limit τ_k and the lower strength limit of the structure τ_r ($\tau_k \approx \tau_r \approx Y$) was used to describe the rheological behavior of the disperse medium. For one-dimensional shear flow the rheological equation has the form [6, 7]

$$\frac{1}{\eta}[\tau - Y\operatorname{sign}(\dot{\gamma})]\cdot 1(|\tau| - Y) + \frac{1}{G}\frac{\partial\tau}{\partial t}(Y - |\tau|) = \dot{\gamma}. \tag{1}$$

Let us examine the strain state of a medium at the time when the structure is destroyed at the channel wall in a narrow zone of width h(t) while the remaining domain is occupied by an elastic structural skeleton (Fig. 1). The assumption made about small h permits finding the flow velocity profile from the destroyed structure by using an inertialess approximation. In this zone

$$v(y, t) = -\frac{1}{2\eta}\left(\frac{\partial p}{\partial z}\right)_0\left[\left(\frac{H}{2}\right)^2 - y^2 + 2\left(\frac{H}{2} - h\right)\left(y - \frac{H}{2}\right)\right], \quad H/2 - h \leqslant y \leqslant H/2. \tag{2}$$

The displacements of a solid skeleton are described analogously [5] by an initial-boundary value problem for the wave equation with boundary conditions dependent on the time and given on the unknown boundary

$$\rho\frac{\partial^2 u}{\partial t^2} = -\left(\frac{\partial p}{\partial z}\right)_0 + G\frac{\partial^2 u}{\partial y^2}; \tag{3}$$

$$u|_{t=0} = 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \quad \left.\frac{\partial u}{\partial y}\right|_{y=0} = 0; \tag{4}$$

$$\text{a)}\quad \text{if } G\left.\frac{\partial u}{\partial y}\right|_{y=\frac{H}{2}} < Y, \text{ then } u|_{y=\frac{H}{2}} = 0; \tag{5}$$

$$\text{b)}\quad \text{if } G\left.\frac{\partial u}{\partial y}\right|_{y=\frac{H}{2}} \geqslant Y, \text{ then } \begin{cases} G\left.\dfrac{\partial u}{\partial y}\right|_{y=\frac{H}{2}-h(t)} = Y, & (6)\\ \left.\dfrac{\partial u}{\partial t}\right|_{y=\frac{H}{2}-h(t)} = -\dfrac{h^2(t)}{2\eta}\left(\dfrac{\partial p}{\partial z}\right)_0. & (6.1)\end{cases}$$

The problem (3)-(6.1) is solved by an operational method [8] after removing the boundary conditions (6), (6.1) given on the moving boundary between the destroyed and undestroyed structure to the channel wall. An unknown velocity is introduced on the boundary of the elastic zone $\Phi(t)$:

$$v|_{y=\frac{H}{2}} = \Phi(t). \tag{7}$$

If the stresses τ_w on the channel wall does not exceed the yield point, then $\Phi(t) = 0$, and the adhesion conditions are satisfied. Otherwise $\Phi(t) \neq 0$, and the elastic skeleton seems to slip along the wall at the velocity $\Phi(t)$ because of near-wall flow from the destroyed structure. The profile of the elastic shifts has the form

$$\bar{u} = \sum_{n=0}^{\infty}\left\{\int_0^{\bar{t}}[\bar{\Phi}(\bar{t} - \bar{t}') + (\bar{t} - \bar{t}')]\left[1\left(\bar{t}' - \frac{4n - (\bar{y} - 1)}{a}\right) - \right.\right.$$

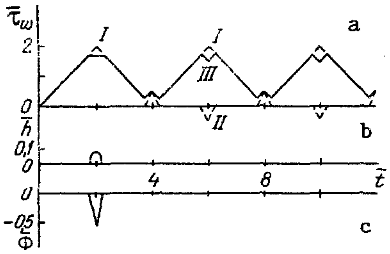

Fig. 2. Dependence of the stress on the channel wall (a), width of the layer with destroyed structure (b), and velocity on the structural skeleton boundary (c) on time for $a = 1$, $\alpha = 25$, $\varepsilon = 0.25$.

$$-1\left(\bar t' - \frac{4n+2-(\bar y-1)}{a}\right) - 1\left(\bar t' - \frac{4n+2+(\bar y+1)}{a}\right) + 1\left(\bar t' - \frac{4n+(\bar y+1)}{a}\right)\Bigg] d\bar t'\Bigg\} - \frac{1}{2}\bar t^{2},$$

where

$$\bar u = \frac{uT_y}{H/2},\quad \bar y = \frac{y}{H/2},\quad \bar\Phi = \frac{\Phi T_y a}{H/2},$$
$$\bar t = \frac{ta}{T_y},\quad a = [G/(H(\partial p/\partial z)_0/2)]^{1/2}. \tag{8}$$

The parameter a is the characteristic of the elastic strains in the medium $\max(\partial u/\partial y) = 2/a^2$.

The function $\bar\Phi(\bar t)$ can be determined from the conditions (6) and (6.1) referred to the channel boundary

$$\sum_{n=0}^{\infty}\left\{\int_0^{\bar t}\left[\frac{d\bar\Phi}{d\bar t}(\bar t-\bar t') + 1\cdot\right]\left[1\left(\bar t' - \frac{4n+\bar h(\bar t)}{a}\right) - \right.\right.$$
$$-1\left(\bar t' - \frac{4n+2+\bar h(\bar t)}{a}\right) + 1\left(\bar t' - \frac{4n+2-\bar h(\bar t)}{a}\right) -$$
$$\left.\left.-1\left(\bar t' - \frac{4n+4-\bar h(\bar t)}{a}\right)\right]d\bar t'\right\} - \bar t = -\alpha\bar h^2,\quad \bar h = \frac{h}{H/2},$$
$$\alpha = \frac{1}{2\eta}\left[\rho\left(\frac{H}{2}\right)^3\left(\frac{\partial p}{\partial z}\right)_0\right]^{1/2}, \tag{9}$$
$$\sum_{n=0}^{\infty}\left\{\int_0^{\bar t}[\bar\Phi(\bar t-\bar t') + (\bar t-\bar t')]\left[\delta\left(\bar t' - \frac{4n}{a}\right) - 2\delta\left(\bar t' - \frac{4n+2}{a}\right) + \delta\left(\bar t' - \frac{4n+4}{a}\right)\right]d\bar t'\right\} = (2-\varepsilon)/a.$$

Therefore, the solution of the problem reduces to two integral relationships (9) for the velocity on the boundary between the destroyed and undestroyed structures and for the width of the viscoplastic flow $\bar h(\bar t)$. Represented in Table 1 and Fig. 2 are dependences of the stress on the channel wall $\tau_w(a)$, the width of the layer from the destroyed structure $\bar h(\bar t)$(b) and the velocity on the structural skeleton boundary $\bar\Phi(\bar t)$ (c) on the time ($a = 1$, $\alpha = 25$, $\varepsilon = 0.25$). Upon imposition of a pressure gradient on the channel boundary a loading wave is generated. The stress τ_w grows linearly. For $\bar t = (2-\varepsilon)/a$ its magnitude reaches the yield point $\bar Y = Y/(H(\partial p/\partial z)_0/2)$ consequently destruction of the structure and slippage of the structural skeleton along the wall start in the near-wall domain. Growth of the absolute value of the slippage velocity ($\bar\Phi(\bar t) \le 0$) diminishes the amplitude of the perturbations being propagated to the channel center as compared with the case of structure strain without destruction. For $\bar t = 2/a$ the modulus of the velocity on the boundary between the domains with the destroyed and undestroyed structure $|\bar\Phi(\bar t)|$ reaches the maximal value and then decreases. For $\bar t > 2/a$ an unloading wave is formed in the structure. Diminution of the magnitude of the velocity of structural skeleton motion (in modulus) now increases the amplitude of the perturbations being separated from the boundary. For $\bar t = (2+\varepsilon)/a$ the stress on the boundary element of the skeleton drops below the yield point, consequently, the domain of the destroyed structure vanishes,

TABLE 1. Change of the Functions $\bar{\tau}_w(t)$, $\bar{\Phi}(t)$, $\bar{h}(t)$ with Time

Interval boundaries	$\bar{\tau}_w(\bar{t})$	$\bar{\Phi}(\bar{t})$	$\bar{h}(\bar{t})$
$0,\ (2-\varepsilon)/a$	$a\bar{t}$	0	0
$(2-\varepsilon)/a,\ \bar{t}_*^{(1)}$	$2-\varepsilon$	$(2-\varepsilon)/a-\bar{t}$	$[(a\bar{t}-2+\varepsilon)/(\alpha a)]^{1/2}$
$\bar{t}_*^{(1)},\ 2/a$	$2-\varepsilon$	»	$[-1+(1+4\alpha\varepsilon a)^{1/2}]/(2\alpha a)$
$2/a,\ \bar{t}_*^{(2)}$	$2-\varepsilon$	$-(2+\varepsilon)/a+t$	»
$\bar{t}_*^{(2)},\ (2+\varepsilon)/a$	$2-\varepsilon$	»	$[(-a\bar{t}+2+\varepsilon)/(\alpha a)]^{1/2}$
$(2+\varepsilon)/a,\ (4-\varepsilon)/a$	$4-a\bar{t}$	0	0
$(4-\varepsilon)/a,\ 4/a$	$-4+2\varepsilon+a\bar{t}$	0	0
$4/a,\ (4+\varepsilon)/a$	$4+2\varepsilon-a\bar{t}$	0	0
$(4+\varepsilon)/a,\ (6-\varepsilon)/a$	$-4+a\bar{t}$	0	0
$(6-\varepsilon)/a,\ 6/a$	$8-2\varepsilon-a\bar{t}$	0	0
$6/a,\ (6+\varepsilon)/a$	$-4-2\varepsilon+a\bar{t}$	0	0
$(6+\varepsilon)/a,\ (8-\varepsilon)/a$	$8-a\bar{t}$	0	0
$(8-\varepsilon)/a,\ 8/a$	$-8+2\varepsilon+a\bar{t}$	0	0

$$\bar{t}_*^{(1)} = 2/a + (1-(1+4\alpha a\varepsilon)^{1/2})/(2\alpha a^2)$$

$$\bar{t}_*^{(2)} = 2/\alpha + (-1+(1+4\alpha a\varepsilon)^{1/2})/(2\alpha a^2)$$

the slippage ceases. As is seen from the solution, the transient is also terminated since periodic stress and displacement vibrations are later established in the medium. The steady stress wave profile (III) differs from the corresponding profile of a wave being propagated in an elastic structure. It is (Fig. 2) the superposition of the triangular waves I and the wave II formed because of formation of flow zones of a medium with destroyed structure. For $\bar{t} = (4 - \varepsilon)/a$ an unloading wave I and a loading wave II approach the wall. Their imposition yields stress growth on the wall. For $\bar{t} = 4/a$ unloading I and II waves stand off the boundary, resulting in a diminution of $\bar{\tau}_w$. For $\bar{t} > (4 + \varepsilon)/a$ the amplitude of the wave II equals zero on the channel boundary. Growth of the tangential stress is determined only by the loading wave I etc. The period of steady vibrations is $T = 4/a$.

It follows from the condition $\bar{h}(\bar{t}) \ll 1$ that, as has been noted earlier, the relationships $\varepsilon \ll 1$, $T_b/T_y \ll 1$ should be satisfied. This last inequality is necessary for inertialless flow of the destroyed structure.

Shown in Fig. 3 are results of a numerical solution of the problem. The asymptotic solution is qualitatively in agreement with the numerical solution that satisfactorily reflects the singularity of the transient and steady shear strain process. The dashed line refers to the analytic, and the solid line to the numerical solution ($a = 1$, $\alpha = 25$, $\varepsilon = 0.5$). The yield point is $1\text{-}10^4$ Pa for many disperse systems. The dynamic transient mode in a 10^{-2} m wide channel is realized for pressure gradients, respectively, of $10^2\text{-}10^5$ Pa/m. The transient terminates at $t = (2 + \varepsilon)T_y$ and its duration is $\sim 1\text{-}10^{-3}$ sec for G from 1 Pa to 10^4 Pa. As the asymptotic solution shows, the undestroyed structure shifts a distance $\varepsilon^2 H/2$ ($\sim 10^{-4}$ m) along the channel in practice during a time interval εT_y ($\sim 10^{-1}\text{-}10^{-4}$ sec) in the dynamic transient mode. The brief slippage of the elastic skeleton relative to the channel walls is related to loading and unloading wave propagation in the undestroyed structure. The structure is destroyed at the wall in the loading phase. The near-wall flow of the medium causes a shift of the undestroyed structure which corresponds to the slippage. Reconstruction of the tangential stress wave profile diminishes its amplitude, consequently, destruction of the structure does not later occur. Therefore, the results obtained show that because of the excess of the dynamic loads in the undestroyed structure ($\tau_{max} = H(\partial p/\partial z)_0$) of the static ($\tau_{max} = H(\partial p/\partial z)_0/2$) a transient mode is possible to accompany moderate progress of the undestroyed structure along the channel. A break in the pressure gradient results in a change in the elastic wave amplitude. The greatest possible stress wave amplitude equals the yield point (for a break in the pressure gradient at the time $\bar{t} = (2 + \varepsilon)/a$). If an additional rise in the pressure gradient by a very small quantity $\sim\varepsilon(\partial p/\partial z)_0$ is realized after the break, then again brief slippage of

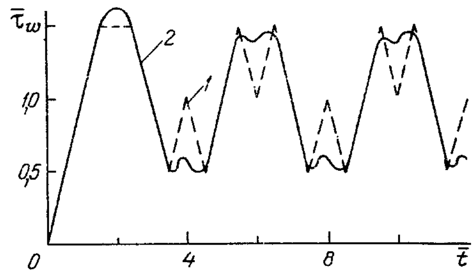

Fig. 3. Comparison of the analytical and numerical [4] solutions for $a = 1$, $\alpha = 25$, $\varepsilon = 0.5$: 1) analytical, and 2) numerical computation.

the skeleton will occur along the channel (the amplitude of superposition of the reverse stress wave III and the wave being generated at the channel wall because of the additional rise in the pressure gradient exceeds the yield point). The motion mechanism for a medium of the type considered can be realized for vibration actions.

NOTATION

τ, shear stress; τ_k, yield point; τ_r, τ_b, lower and upper bounds of the plastic flow domain; H, channel width; $\partial p/\partial z$, pressure gradient; h, width of the flow zone with destroyed structure; $1(x)$, Heaviside function; G, elastic modulus of the structural skeleton; η, plastic viscosity; Y, yield point; $\dot{\gamma}$, shear velocity; $v = \partial u/\partial t$, velocity of particles of the medium; u, particle displacement; y, z, Cartesian coordinates, and τ_w, shear stress on the channel wall.

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